Short Papers

A Broad-Band Element for Microstrip Bias or Tuning Circuits

B. A. SYRETT

Abstract—**A microstrip radial transmission line circuit element, namely a 180° circular stub or "half-moon" structure, having a reflection coefficient of unity magnitude and phase which varies slowly with frequency is presented. Theoretical reflection coefficient data are shown to agree well with experimental microstrip ($\epsilon_r = 2.35$, $h = 250$ μm) data in X band. Applications of the half-moon microstrip element in the design of broad-band bias or tuning networks are discussed.**

I. Introduction

An efficient means of supplying dc bias to active devices is an important consideration in the design of amplifying or oscillating circuits. A typical bias network comprises a dc blocking capacitor, and an RF bias line in the form of a low-pass filter or choke. Since the biasing must be achieved with minimum RF energy loss to the bias line and dc source, the input impedance of the bias line should be an RF open-circuit or an RF short-circuit. For active devices which must operate over a bandwidth greater than say 100 MHz, the designed RF open- or RF short-circuit must also be broad band. Bias lines constructed using lumped elements [1] or sections of quarter-wavelength transmission lines [2] may not have sufficient bandwidth for such applications. This paper concerns the analysis and experimental characterization of a radial microstrip element which is suitable for broad-band bias or tuning networks.

II. Theory

The microstrip configuration under investigation is a radial transmission-line element in the form of a circular stub with vertex angle α, inner radius r_i, and outer radius r_L as shown in Fig. 1(a), separated from a ground plane by a dielectric of relative permittivity ϵ_r, permeability μ_0, and thickness h as shown in Fig. 1(b). The input signal is assumed to be uniformly applied along the inner periphery at radius r_i. Exact analysis of the input impedance of the microstrip element is impossible since the mode of propagation is not a pure radial TE or TM mode. However, an approximate analysis can be undertaken as follows.

First consider the angle α to be 2π so that the microstrip element becomes a ring resonator. A magnetic-wall model [3] can be employed for analysis where the ring is considered as a cavity resonator with electric walls on the upper conductor and bottom ground plane, and magnetic walls at radii r_i and r_L. Thus it is assumed that the electromagnetic fields are confined entirely inside the dielectric under the upper conductor ring pattern. The oscillation modes of this resonator model which have a field distribution similar to that of the quasi-TEM mode of the input microstrip line are the z-independent modes (the z axis is perpendicular to the plane of the microstrip as shown in Fig. 1(b)). In general, these modes have a longitudinal electric field component E_z, radial magnetic field H_r, and azimuthal magnetic field H_ϕ as shown in [3].

Manuscript received August 31, 1979; revised March 19, 1980.
The author is with the Department of Electronics, Carleton University; Ottawa, Canada K1S 5B6.

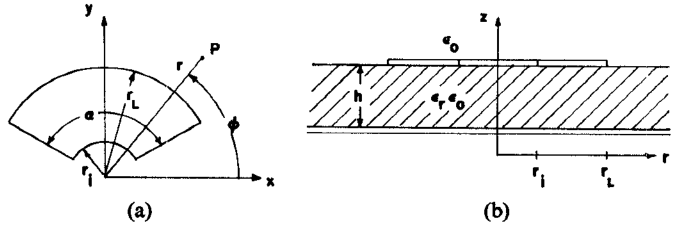

Fig. 1. (a) Geometry of the radial microstrip transmission-line element. (b) Cross section of the microstrip transmission line.

The lowest order mode that can be excited is the radial transmission-line mode. This mode has no field variations in the ϕ direction, no field components in the radial direction, but has field components E_z and H_ϕ only, given by [4], [5]

$$E_z(r) = AH_0^{(1)}(kr) + BH_0^{(2)}(kr) \tag{1a}$$

and

$$H_\phi(r) = \frac{-k}{j\omega\mu_0}\left[AH_1^{(1)}(kr) + BH_1^{(2)}(kr)\right] \tag{1b}$$

where $H_n^{(1)}$ is the Hankel function of the first kind of order n, $H_n^{(2)}$ is the Hankel function of the second kind of order n, and $k = \omega\sqrt{\epsilon_r\epsilon_0\mu_0}$ is the wavenumber. The $H_n^{(1)}$ terms represent waves travelling radially inward while the $H_n^{(2)}$ terms represent waves travelling radially outward. The Hankel functions may also be written as

$$H_n^{(1)}(kr) = J_n(kr) + jN_n(kr) \tag{2a}$$

$$H_n^{(2)}(kr) = J_n(kr) - jN_n(kr) \tag{2b}$$

where J_n is the Bessel function of the first kind of order n, and N_n is the Bessel function of the second kind of order n. The constants A and B in (1) may be determined by specifying an input wave impedance $Z_i = E_z(r_i)/H_\phi(r_i)$ when load wave impedance $Z_L = E_z(r_L)/H_\phi(r_L)$ is given. The input wave impedance is [4]

$$Z_i = Z_{0i}\left[\frac{Z_L\cos(\theta_i - \psi_L) + jZ_{0L}\sin(\theta_i - \theta_L)}{Z_{0L}\cos(\psi_i - \theta_L) + jZ_L\sin(\psi_i - \psi_L)}\right] \tag{3}$$

where Z_0 is the wave impedance of a radial transmission line given by

$$Z_0(kr) = \sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}\left[\frac{J_0^2(kr) + N_0^2(kr)}{J_1^2(kr) + N_1^2(kr)}\right]^{1/2} \tag{4}$$

and angles θ and ψ are given by

$$\theta(kr) = \arctan\left[N_0(kr)/J_0(kr)\right] \tag{5a}$$

$$\psi(kr) = \arctan\left[-J_1(kr)/N_1(kr)\right]. \tag{5b}$$

The subscripts i or L on the parameters Z_0, θ, and ψ in (3) denote evaluation of these parameters at radius r_i or r_L, respectively.

For a radial transmission line having $r_i < r_L$, the total voltage between the conductors is $-E_zh$; the total radial current out-

0018-9480/80/0800-0925$00.75 ©1980 IEEE

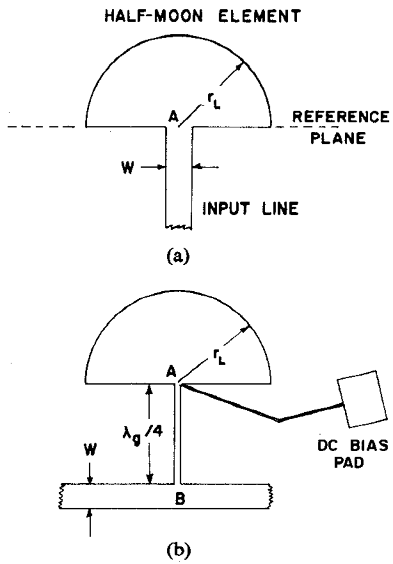

Fig. 2. (a) The half-moon element-end terminating an input microstrip transmission line. (b) Use of the half-moon element to provide a broad-band RF open-circuit at point B for biasing shunt-mounted active devices.

ward in the microstrip conductor and inward on the ground plane is $2\pi r H_\phi$. Thus the input terminal impedance Z_T is [4]

$$Z_T = -\frac{h}{2\pi r_i}\frac{E_z(r_i)}{H_\phi(r_i)} = -\frac{h}{2\pi r_i}Z_i \tag{6}$$

for the ring resonator.

The microstrip element under consideration here is an open-circuited (ideally $Z_L = \infty$) radial element with angle α less than 2π. As a first approximation it may be assumed that the terminal impedance is uniformly distributed along the periphery at radius r_i. Then from (3) and (6) the input impedance of an open-circuited microstrip radial element with vertex angle α radians is [6], [7]

$$Z_T = j\frac{hZ_0(kr_i)\cos[\theta(kr_i) - \psi(kr_L)]}{\alpha r_i \sin[\psi(kr_i) - \psi(kr_L)]}. \tag{7}$$

For stripline elements there are two such impedances in parallel resulting in one-half the terminal impedance of a microstrip element. Edge effects of the microstrip element, which are not taken into account by the magnetic-wall model, can be described approximately by replacing relative permittivity ϵ_r by the effective permittivity ϵ_{eff} defined by Wheeler [8].

In practice, the microstrip element end-terminates an input microstrip line of width W and impedance Z_0 related as given in Wheeler's curves [8]. The resulting reflection coefficient Γ at the terminal impedance Z_T has unit magnitude and arbitrary phase arg (Γ). The radius r_i is chosen as $W/2$; radius r_L is determined by the required center frequency of operation f_0 and phase arg (Γ_0) at that frequency; the angle α is determined from bandwidth requirements. For bandwidth considerations the change in arg (Γ) was investigated as frequency was changed for various α with r_i and r_L fixed. The computed results indicate that as the angle α is increased, the variation in arg (Γ) over 8–12 GHz decreases. For example, if $r_i = 0.38$ mm and $r_L = 4$ mm, the variation in arg (Γ) over this band is 39° for $\alpha = \pi/2$, 26° for $\alpha = 2\pi/3$, 20° for $\alpha = \pi$, 16° for $\alpha = 5\pi/4$, and 14° for $\alpha = 3\pi/2$. To avoid circuit layout problems and also possible adverse junction effects while maintaining reasonably broad bandwidth, the angle $\alpha = \pi$ is suitable, resulting in a "half-moon" microstrip element as shown in Fig. 2(a).

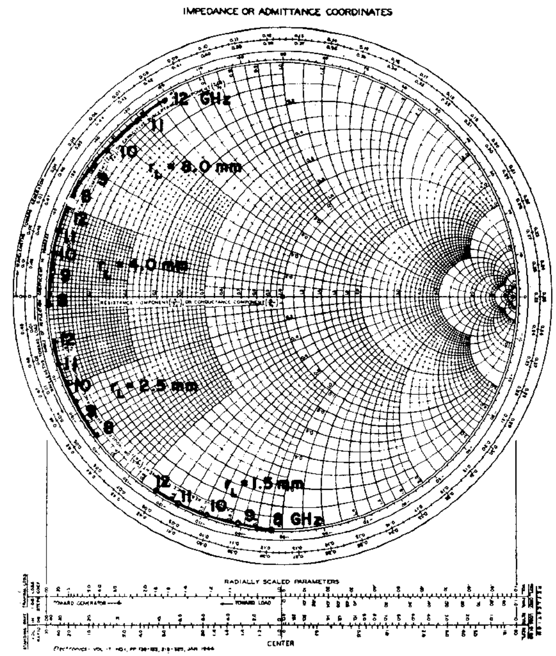

Fig. 3. The measured input impedance in X band of half-moon elements of various radii terminating a 50-Ω input microstrip transmission line ($\epsilon_r = 2.35$, $h = 250$ μm).

III. Experimental Results

Ten half-moon radial line elements with outer radius in the range $1.5 \leqslant r_L \leqslant 9.0$ mm were fabricated on Duroid D-5870 ($h = 250$ μm, $\epsilon_r = 2.35$) substrate.[1] The input transmission line had 50-Ω characteristic impedance with $W = 760$ μm, so that $r_i = 380$ μm for all circuits. The circuits were tested in X band using a Hewlett Packard Automatic Network Analyser operating in reflection mode. The reference plane was established at point A in the circuit of Fig. 2(a) using three microstrip calibration transmission lines in conjunction with a computer program [9].

The reflection coefficient Γ measured at the reference plane is plotted in Fig. 3 for several values of r_L. The results to be noted are the following:

1) $|\Gamma| \simeq 1.0$ for all radii $1.5 \leqslant r_L \leqslant 9.0$ mm over the frequency band 8–12.4 GHz (estimated measurement error was ± 0.02)
2) arg (Γ) changed only $\pm 15°$ over the 8–12.4-GHz band about a phase value varying from $+250°$ at center frequency 10 GHz for $r_L = 1.5$ mm to $+140°$ at 10 GHz for $r_L = 8.0$ mm (estimated measurement error was $\pm 2°$).

The measured dependence of arg (Γ) on the radius r_L of the half-moon with frequency as a parameter is shown in Fig. 4; theoretical behavior calculated using (7) is also plotted. Fig. 4 shows very good agreement between theoretical and experimental values of arg (Γ) for $2 < r_L < 9$ mm and frequency in X band.

III. Applications

The radial line element with $\alpha < \pi$ has been used as a resonator section in strip line transmission filters [6], [7]. However, the half-moon element ($\alpha = \pi$) has broadband reflection properties which make it an especially useful one-port microstrip circuit. One possible application is a broad-band tuning circuit with

[1]Trade name of Rogers Corporation, Rogers CT.

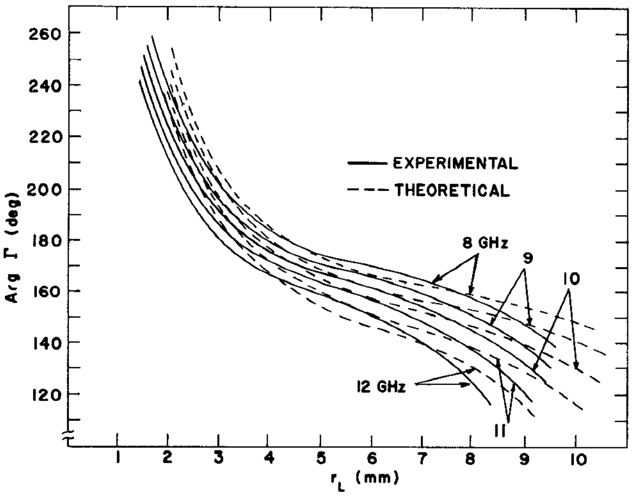

Fig. 4. Comparison of experimental and theoretical results for phase of the reflection coefficient of a half-moon element terminating a 50-Ω input microstrip transmission line ($\epsilon_r = 2.35$, $h = 250$ μm) as a function of radius with X-band frequency as a parameter.

specified reactance (approximately zero resistive component) at the center frequency as determined from information in Fig. 3. For example if a load with −24-Ω reactance at 10 GHz is required, this corresponds to $\Gamma = 1.0\angle -129°$ in a 50-Ω system. From Fig. 4 a half-moon circuit of radius $r_L = 1.9$ mm will have the desired reactance.

Also, in Fig. 4 it can be seen that for $2.7 < r_L < 4.3$ mm, the half-moon element is a broad-band RF short-circuit at point A with center frequency in X band. Note that since the half-moon circuit is a radial line element, the required r_L is less than a quarter-wavelength at the center frequency. The half-moon circuit can be used as a bias line for series-mounted active devices with the dc bias applied at point A through a microstrip transmission line of high characteristic impedance (> 100 Ω). Typically the arg (Γ) changes only ±15°, about the phase 180° at center frequency, over a 6-GHz bandwidth.

Alternative RF short-circuits have smaller bandwidth and larger area. For example, an RF short-circuit designed at 10 GHz using the half-moon element was compared with a low-impedance open-circuited quarter-wavelength microstrip stub. From Fig. 4, an outer radius $r_L = 3.65$ mm is required for the half-moon circuit (with inner radius $r_i = 0.38$ mm corresponding to a 50-Ω input microstrip line). For a meaningful comparison the characteristic impedance of the microstrip stub must be chosen carefully. A terminal characteristic impedance for the radial line can be defined as

$$Z_{0T}(kr) = \frac{h}{\alpha r} Z_0(kr) \tag{8}$$

from (4) and (7). Then the radial line stub will have an equivalent characteristic impedance defined as

$$Z_{eq} = \sqrt{Z_{0T}(kr_i) Z_{0T}(kr_L)} \, . \tag{9}$$

Since $Z_{0T}(kr_i) = 17.3$ Ω and $Z_{0T}(kr_L) = 5.1$ Ω, a microstrip stub having characteristic impedance $Z_0 = Z_{eq} = 9.4$ Ω was constructed for comparison purposes. The measured arg (Γ) for the 9.4-Ω open-circuited quarter-wavelength stub changed 29° over the band 7–13 GHz whereas the arg (Γ) for the half-moon element changed 22° over the same bandwidth. In addition to the wider bandwidth, the half-moon element has a smaller physical area (21 mm^2) than the low-impedance rectangular stub (28 mm^2).

It is usually advantageous for shunt-mounted active devices to be biased looking into an RF open-circuit. The broad-band RF short-circuit at point A in Fig. 2(b) can be transformed into an RF open-circuit at point B, with little sacrifice in bandwidth, by means of a quarter-wavelength microstrip line of high characteristic impedance. Again the dc bias should be applied at point A through another line of high characteristic impedance but of arbitrary length. Typically arg (Γ) at point B changed ±18°, about phase 0° at center frequency, over 4-GHz bandwidth. Two such configurations can be cascaded to improve bandwidth if necessary.

V. Conclusions

The half-moon radial line element displays broad-band reflection coefficient properties which make it a useful one-port microstrip circuit element. It is especially well suited for use in broad-band RF bias lines or tuning circuits for active devices. The analytical model presented for the half-moon circuit can be used for computer-aided circuit design.

Acknowledgment

The author would like to thank L. Besser of COMPACT Engineering for his helpful correspondence and also the Microwave Institute in Stockholm, Sweden, for providing the measurement facilities used during the course of this work.

References

[1] C. S. Aitchison *et al.*, "Lumped-circuit elements at microwave frequencies," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 928–937, Dec. 1971.
[2] R. V. Garver, "360° varactor linear phase modulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 137–147, Mar. 1969.
[3] I. Wolff and N. Knoppik, "Microstrip ring resonator and dispersion measurement on microstrip lines," *Electron. Lett.*, vol. 7, pp. 779–781, Dec. 30, 1971.
[4] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1967, pp. 453–458.
[5] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits*, (MIT Rad. Lab. Series, no. 8). New York: McGraw-Hill, 1948.
[6] J. P. Vinding, "Radial line stubs as elements in stripline circuits," *NEREM Record*, pp. 108–109, 1967.
[7] J. P. Vinding, "Strip line electrical filter element," U. S. Patent 3 566 315, Feb. 1971.
[8] H. A. Wheeler, "Transmission-line properties of parallel strips separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172–185, Mar. 1965.
[9] B. A. Syrett, "The use of the automatic network analyzer in the development and modelling of a novel broad-band bias line for X-band microstrip circuits," M. Eng. thesis, Carleton University, Ottawa, Canada, Jan. 1973.

Dispersion in *n* Coupled Microstrip Meanders

A. K. AGRAWAL

***Abstract*—A meander line consisting of an even (n), or odd ($n-1$), number of coupled microstrips has been analyzed for its dispersion and iterative impedance characteristics. In contrast with the unit cell approximation used by other authors, this method takes all the couplings into account and enables correct determination of stopband locations, which is**

Manuscript received October 23, 1979; revised March 19, 1980.
The author is with Panjab Engineering College, 323/15-A Chandigargh Pin 160016, India.